

Consider a system  $C$  that can be structured as  $S+E$  or  $S'+E'$ , where every subsystem ( $S, S', E, E'$ ) is defined by a proper set of degrees of freedom. Quantum mechanically, different structures of  $C$  ( $S+E = C = S'+E'$ ) pertain to different tensor-product-structures (TPSs) of the system's Hilbert state space.

While "structures" ( $S+E$ , or  $S'+E'$ , or...), i.e. the TPSs, are mutually linked by some linear canonical transformations (LCTs), **there is not any transformation that should be applied to a quantum state or to an observable of the system.**

The task is to link the different forms of a quantum state or of an observable (hermitean operator) of the total system for a pair of structures. In other words: there is a unique state or an observable [in a given instant of time] and the task is to write down or at least qualitatively consider different forms of the state or of the observable relative to the chosen structure(s). Hence

**No transformation of the state or of the composite system's observable is allowed.**

**Why?**

**Because it changes the state or the observable of interest.**

**Rather: keep the state (the observable) fixed and relate different forms of the state (of the observable) for the chosen TPSs.**

**The TPSs bring about the elementary concept of quantum locality (of systems and operations).**

**Doing otherwise is a misconception or at least not the "quantum structures" we're interested in.**

**EXAMPLE:** Consider the hydrogen atom structured as *electron--proton* ( $e+p$ ) or as *center-of-mass--relative-particle* ( $CM+R$ ). The atom's Hilbert space is factorized:  $H_e \times H_p$  or  $H_{CM} \times H_R$ . Notice:  $H_e \times H_p = H_R \times H_{CM}$ . Then [neglecting the particles spin]:

$$|nlm\rangle_R \times |\omega\rangle_{CM} = |\varphi\rangle = \sum_i c_i |i\rangle_e \times |i\rangle_p$$

$$T_{CM} \times I_R + I_{CM} \times \left( T_R - \frac{1}{4\pi\epsilon_0} \frac{e^2}{r_R} \right) = H = T_e \times I_p + I_e \times T_p - \frac{1}{4\pi\epsilon_0} \frac{e^2}{|\vec{r}_e \times I_p - I_e \times \vec{r}_p|}$$

where  $|\varphi\rangle$  and  $H$  represent the atom's quantum state and the Hamiltonian, respectively. It is essential to note that the most-left and the most-right hands of the above equations are not obtained via any transformations performed **on** the state or **on** the Hamiltonian – both [state and Hamiltonian] are **fixed**. But the **forms** of the state and of the Hamiltonian are different for the chosen structures of the atom,  $CM+R$  and  $e+p$ , respectively. Every subsystem,  $CM, R, e$  and  $p$ , is "local system".

**Everything equally applies to every single state and observable of any quantum-mechanical system.**